CORRIGENDUM TO PROPOSITION 3.1 IN "L^∞-NORMS OF EIGENFUNCTIONS FOR ARITHMETIC 3-MANIFOLDS"

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In the above paper published in Duke Math. J. **77** (1995) 799-817, Proposition 3.1 as well as its proof should be corrected.

We follow the modification done in [1]. We first have to supply the factor r_j^ε in Lemma 3.2. The corrected statement is

Lemma 3.2' Under the normalization $\|\phi_j\|_2 = 1$, we have

$$\sum_{|n|<N} |c_j(n)|^2 \ll e^{\pi r_j} \left(r_j + \frac{N^2}{r_j} \right) r_j^\varepsilon.$$

Then Proposition 3.1 is replaced by the following:

Proposition 3.1' Let ϕ_j be a cusp form as above with Laplace eigenvalue $\lambda_j = 1 + r_j^2$. Then

$$|\phi_j(v)| \ll r_j^\varepsilon \left(\frac{r_j}{y} + \frac{r_j^{1/2}}{y^{1/3}} \right) \|\phi_j\|_2.$$

Proof. We estimate the Fourier series (3.1):

$$\phi_j(v) = \sum_{n \in O} c_j(n) y K_{ir_j}(2\pi|n|y) e(\langle n, x\rangle).$$

We use the well-known bounds

$$K_{ir}(2\pi|n|y) \ll |(|n|y)^2 - r^2|^{-\frac{1}{4}} e^{-\frac{\pi}{2}r}, \tag{A}$$

$$K_{ir}(2\pi|n|y) \ll r^{-\frac{1}{3}} e^{-\frac{\pi}{2}r}, \tag{B}$$

$$K_{ir}(2\pi|n|y) \ll (|n|y)^{-\frac{1}{2}} e^{-|n|y}. \tag{C}$$

Applying (C) for $|n| \geq r_j/y$ to get

$$\sum_{|n|\geq r_j/y} |c_j(n) y K_{ir_j}(2\pi|n|y)| \ll O(e^{-r_j}).$$

Thus we estimate by putting $N = r_j/y$

$$\begin{aligned}\phi_j(v) &\ll \sum_{|n|<N} |c_j(n) y K_{ir_j}(2\pi|n|y)| + O(e^{-Ny}) \\ &\ll y \left(\sum_{|n|<N} |c_j(n)|^2\right)^{1/2} \left(\sum_{|n|<N} |K_{ir_j}(2\pi|n|y)|^2\right)^{1/2} + O(e^{-Ny}) \\ &\ll r_j^{\frac{1}{2}+\varepsilon} e^{\pi r/2} \left(\sum_{|n|<N} |K_{ir_j}(2\pi|n|y)|^2\right)^{1/2} + O(e^{-Ny})\end{aligned}$$

from Lemma 3.2'. By (A), the partial sum over $|n| < N-1$ is bounded by the integral as

$$\begin{aligned}e^{\pi r_j} \sum_{|n|<N-1} |K_{ir_j}(2\pi|n|y)|^2 &\ll \sum_{|n|<N-1} (r_j^2 - (|n|y)^2)^{-\frac{1}{2}} \\ &\le \int_0^{2\pi} \int_0^{N-1} (r_j^2 - R^2 y^2)^{-\frac{1}{2}} R dR d\theta \ll \frac{r_j}{y^2}\end{aligned}$$

with n substituted by a complex number $Re^{i\theta}$. On the other hand by using (B) for the neighborhood of $|n| = N$, we compute

$$e^{\pi r_j} \sum_{||n|-N| \le r_j^{2/3}/y} |K_{ir_j}(2\pi|n|y)|^2 \ll \sum_{||n|-N| \le r_j^{2/3}/y} r_j^{-\frac{2}{3}} \ll \frac{r_j^{2/3}}{y^2} + \frac{1}{y^{2/3}},$$

because the number of n's participating in the sum is

$$\left(N + \frac{r_j^{2/3}}{y}\right)^2 - \left(N - \frac{r_j^{2/3}}{y}\right)^2 + O\left(N^{2/3}\right) \ll \frac{r_j^{4/3}}{y^2} + \frac{r_j^{2/3}}{y^{2/3}}.$$

Therefore

$$\phi_j(v) \ll r_j^{\frac{1}{2}+\varepsilon} \left(\frac{r_j^{1/2}}{y} + \frac{r_j^{1/3}}{y^{1/3}}\right) = r_j^{\varepsilon} \left(\frac{r_j}{y} + \frac{r_j^{1/2}}{y^{1/3}}\right).$$

□

Remark. This change does not give any influence on the main theorem. All we should do is to apply Proposition 3.1' for $y \ge r_j^{5/19}$ and the other estimate $|\phi_j(v)|^2 \ll \lambda_j^{\frac{37}{38}+\varepsilon} + y^2 \lambda_j^{\frac{27}{38}+\varepsilon}$ in page 816 for $y \le r_j^{5/19}$.

References

[1] P. Sarnak, *Letter to Morawetz.* (availble at http://publications.ias.edu/sarnak)